

# Deep Reinforcement Learning in a Monetary Model

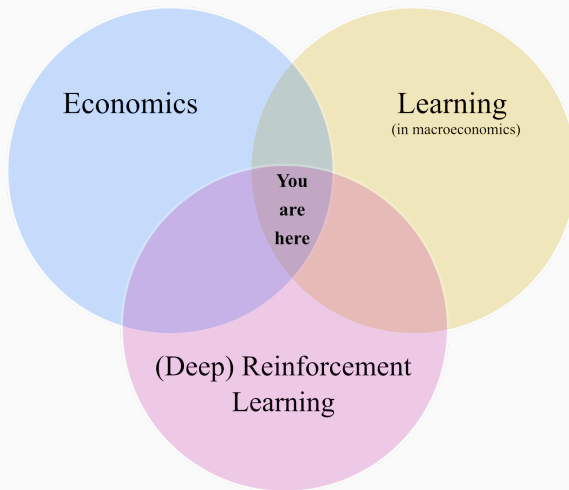
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## Overview & motivation

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- Rational Expectation (RE) convenience choice to solve a model, but not necessarily how people and businesses actually behave
- Learning approach to **bounded rationality (BR)**: specify agent knowledge and behaviour away from RE (often ad hoc)
- BR broadens available state space. See also Moll (2024).
- Example: Monetary policy reaction functions possibly very different under learning, such as forward guidance or the stability of Taylor rules

See Eusepi and Preston (2018) and Hommes (2021) for recent reviews.

## Example: Adaptive learning

Agents are “econometricians” trying to estimate expected quantities via

$$x_{t+1}^e = x_t^e + \phi_t(x_{t-1} - x_t^e), \quad (1)$$

with a gain series  $\phi_t$ .

Under least-squares learning it is usually taken to be  $1/t$ . Together with the (optimal) behavioural rules, i.e. linearised FOCs, this leads to a set of ordinary differential equations determining the expectations (E-)stability of the model.

That is, if a steady state is **stable under learning**, which then serves as a selection criterion.

See, Sargent (1993) and Evans and Honkapohja (2001).

Models populated with *Adaptively Learning Agents* put the agents on an equal footing with the econometrician who is observing data from the model.

- However, this type of *parametric* recursive method assumes that agents correctly specify the laws of motion and other relevant functional relationships of the model

We work with models populated by *Deep Reinforcement Learning Agents (a.k.a. Artificially Intelligent Agents)* who

- have no a priori knowledge about the structure of the economy
- use their utility realisations in response to their actions in order to learn nonlinear decision rules via deep artificial neural networks

We adopt a policy-based deep reinforcement learning approach that can deal with high dimensional continuous action spaces.

Our approach enables agents to learn flexibly, as our learning algorithms are *nonparametric* and recursive, reducing the risk of misspecification

Allowing for misspecification and learning via expelling rational expectation agents and replacing them with “artificially intelligent” ones is also **reminiscent Sargent (1993)**



# Applications of Deep (Reinforcement) Learning in macroeconomics

- Global solution technique with no need of linearisation or other approximations
- Principled way to (bounded) rationality, i.e. agent behaviour and knowledge (this paper)
- General approach to heterogeneity, e.g. household income or age distribution (Hill et al., 2021).

⇒ Loads of potential applications in (macro)economics!

# Deep Reinforcement Learning (a.k.a. Artificial Intelligence)

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# DRL at centre of recent advances in Artificial Intelligence

## ARTICLE

doi:10.1038/nature16961

### Mastering the game of Go with deep neural networks and tree search

David Silver<sup>1</sup>\*, Aja Huang<sup>1\*</sup>, Chris J. Maddison<sup>1</sup>, Arthur Guez<sup>1</sup>, L. Julian Schrittwieser<sup>1</sup>, Ioannis Antonoglou<sup>1</sup>, Veda Panneershelvam<sup>1</sup>, John Nham<sup>1</sup>, Nal Kalchbrenner<sup>1</sup>, Ilya Sutskever<sup>2</sup>, Timothy Lillicrap<sup>1</sup>, Thore Graepel<sup>1</sup> & Demis Hassabis<sup>1</sup>

### Playing Atari with Deep Reinforcement Learning

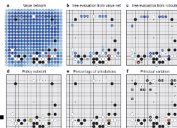
AUTOMATE EXPLORE CUSTOMIZE

**STRONG**  
Carry and power up to 14kg of impact on equipment.

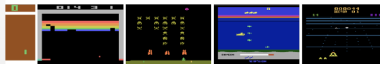
**EASY TO CONTROL**  
Control the robot from afar using an intuitive tablet application and built-in stereo cameras.



**SMART**  
Program repeatable autonomous missions to gather consistent data.



for Mnih Koray Kavukcuoglu David Silver Alex Graves Ioannis Antonoglou  
Daan Wierstra Martin Riedmiller

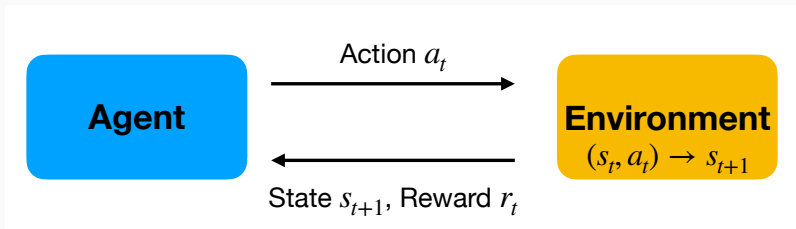


reen shots from five Atari 2600 Games: (Left-to-right) Pong, Breakout, Space Invaders, and Asterix.

Sources: Nature, arXiv, Boston Dynamics



# The Reinforcement Learning (RL) setting



1. Agent observes state of the world  $s_t$
2. Agent takes actions  $a_t(s_t)$
3. Agent receives reward  $r_t$  from environment
4. Actions and state lead to state transition of the environment  $s_{t+1}$

This setting is very general. See Sutton and Barto (2018) for a comprehensive introduction.

## Formal RL definition

The agent aims to maximise expected cumulative lifetime reward, or **expected return**,

$$\max_{\mathcal{P}} \mathbb{E}_t[G_t] \quad \text{with} \quad G_t \equiv \sum_{k=0}^{\infty} \beta^k r_{t+1+k}(s), \quad (2)$$

following a **behavioural policy**  $\mathcal{P} : s_t \rightarrow a_t$ , with  $s_t \in \mathcal{S} \subset \mathbb{R}^{n_s}$  (state space) and  $a_t \in \mathcal{A} \subset \mathbb{R}^{n_a}$  (action space).

The **environment** the agents interaction with returns a reward and a new state, i.e.  $\mathcal{E} : (s_t, a_t) \rightarrow (s_{t+1}, r_t)$ , with  $r_t \in \mathbb{R}$ .

The **state transitions** is modelled as a Markov decision process (MDP)  $\mathcal{T} : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \rightarrow \text{Pr}(s_{t+1}|s_t, a_t) \in [0, 1]$ .

**Problem:** Writing down  $\mathcal{T}$  is simple, knowing  $\mathbb{E}_t[G_t]$  and  $\text{Pr}(s_{t+1}|s_t, a_t)$  is hard (dynamic programming, value function iteration, etc.).

## State and action values

The expected return is maximised by finding the policy  $\mathcal{P}^*$ , which maximises the **values function**

$$V_{\mathcal{P}}(s) = \max_{a \in \mathcal{A}} \mathbb{E}_{\mathcal{P}} [G_t | s = s_t, a = a_t] \quad (3)$$

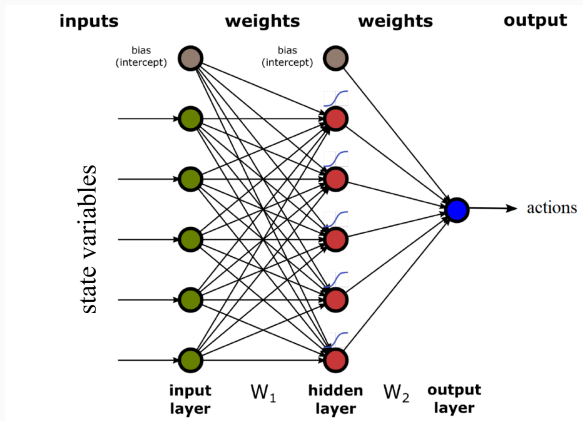
$$= \max_{a \in \mathcal{A}} Q(s, a), \quad (4)$$

with  $Q(s, a)$  the **state action value function**. We are done if we know  $\mathcal{P}^*$  and  $V^*/Q^*$ .

There are different ways to address this problem, which is an area of active AI research.

# Deep Learning + Reinforcement Learning = DRL

In DRL, functions  $\mathcal{P}$  and  $V/Q$  are parameterised using **deep artificial neural networks** (Goodfellow et al., 2016), i.e. neural nets with several hidden layers,  $\mathcal{P}_\phi$  and  $Q_\theta$ :



$\mathcal{P}$  and  $Q$  fulfil the **Bellman equation**

$$Q(s_t, a_t) = r(s_t) + \beta \mathbb{E}_{\mathcal{P}} [Q(s_{t+1}, a_{t+1})]. \quad (5)$$

using sampled state transitions as observations, i.e. interactions of the agent and the environment, and standard optimisation techniques like stochastic gradient descent, the policy and action-value function networks can be trained by iteratively minimising the Bellman residuum,

$$L(\phi, \theta) = \mathbb{E}_{s_t, a_t, r_t} \left[ \frac{1}{2} (Q_{\theta}(s_t, a_t) - \hat{Q}_{\theta}(s_t, a_t))^2 \right], \quad (6)$$

$$\text{with } \hat{Q}_{\theta}(s_t, a_t) = r_t(a_t, s_t) + \beta \mathbb{E}_{\mathcal{P}} [Q_{\theta}(s_{t+1}, \mathcal{P}_{\phi}(s_{t+1}))]. \quad (7)$$

We use Haarnoja et al. (2018). The code we used for optimisation is available at <https://github.com/pranz24/pytorch-soft-actor-critic>.



## General DRL setting for (macro)economics

- Write down model (environment and state)
- Specify **learning** agents, e.g. households, firms, etc., and their **actions**
- Specify state transitions as MDP
- Learning using DRL algorithm (e.g. Haarnoja et al. (2018)):
  1. sample state transition(s) and store in memory
  2. train  $\mathcal{P}_\phi$  and  $Q_\theta$  from memory
  3. test  $\mathcal{P}_\phi$  and  $Q_\theta$  with new state transitions and metric of choice

# Household learning protocol

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**Algorithm 1** Training and testing protocol of household agent

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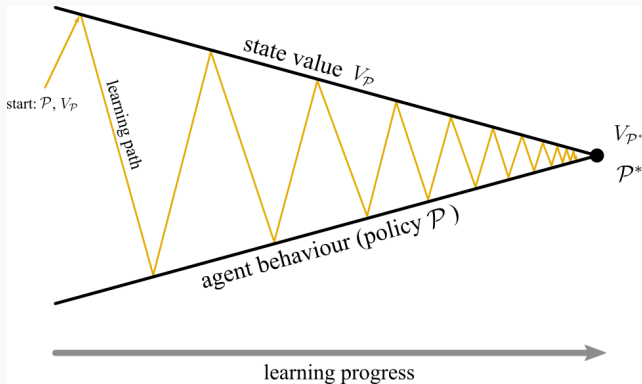
Initialise: Environment  $\mathcal{E}$  (parameterised model), agent (parameterised by  $\mathcal{P}_\phi$ ,  $Q_\theta$ )

```
for steps = 1 to  $N_{train}$  do
  initialise training episode with random state  $s_t$ 
  while training episode is not done do
    if steps  $\leq N_{burn}$  then
      Take allowed random action  $a_t$ 
    else
      Draw exploration action  $a_t = \mathcal{P}_\phi^{exp}(s_t)$ 
    end if
    Environment returns  $(r_t, s_{t+1}) = \mathcal{E}(s_t, a_t)$ 
    Add transition  $(s_t, a_t, r_t, s_{t+1})$  to memory
    Update  $\mathcal{P}_\phi$ ,  $Q_\theta$  using batch gradient descent from memory
    if  $mod(\text{steps}, N_{interval}) = 0$  then
      for test episode = 1 to  $N_{test}$  do
        Record state transitions (*)
      end for
      Save current agent  $(\mathcal{P}_\phi^{steps}, Q_\theta^{steps})$ 
    end if
    State update  $s_t \leftarrow s_{t+1}$ 
    Test episode termination criteria  $(N_{epi}^{max}, d_u^{min})$ 
  end while
end for
Save final agent  $(\mathcal{P}_\phi^{final}, Q_\theta^{final})$ 
```

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# Generalised policy iteration (GPI)

GPI connects economics and learning, and conventional learning approaches with RL



$V^*$ : steady state values,  $\mathcal{P}^*$ : FOC.

## Examples of RL in economics and finance - a very selective literature

- Charpentier et al. (2020): Brief introduction to RL in a economics and finance background
- Zheng et al. (2020): Learning in large-scale geographic ABM
- Calvano et al. (2020): Investigate algorithmic collusion in financial markets
- Chaudhry and Oh (2020): Extract high-frequency expectations in financial markets to measure information effects
- Castro et al. (2021): Learn policy rules of banks participating in a high-value payments system

# The Model Environment

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A single representative household maximises its expected lifetime utility, subject to an inter-temporal budget constraint:

$$\max_{c_t, m_t, n_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, m_t, n_t) \quad \text{s.t.} \quad (8)$$

$$M_t + B_t + C_t = M_{t-1} + B_{t-1}R_{t-1} + W_t n_t - P_t \tau_t, \quad (9)$$

with  $P_t$  the price level at time  $t$ ,  $x_t = \frac{X_t}{P_t}$ ,  $x \in \{M_t, B_t, C_t, W_t\}$  relate real and nominal money, government bonds, consumption and wages, and  $\tau_t$  is a real lump-sum tax to the government each period.

We take the utility (Evans and Honkapohja, 2005)

$$U(c_t, m_t, n_t) = \frac{c_t^{1-\sigma}}{1-\sigma} + \chi \frac{m_t^{1-\sigma}}{1-\sigma} - \frac{n_t^{1+\varphi}}{1+\varphi}. \quad (10)$$

A single representative firm produces according to

$$y_t = \varepsilon_t^y n_t, \quad (11)$$

with technology (shock)  $\varepsilon_t^y$ , maximising profits

$$\max_{w_t} y_t - w_t n_t, \quad (12)$$

by setting setting the optimal wage

$$w_t = \varepsilon_t^y. \quad (13)$$

Markets clear every period, i.e.

$$y_t = c_t \quad (\text{goods}), \quad (14)$$

and

$$c_t^\sigma n_t^\varphi = \varepsilon_t^y \quad (\text{labour}). \quad (15)$$



The government issues interest-bearing bonds and non-interest-bearing currency (money), and collects taxes under the real inter-temporal *government budget constraint* (GBC)

$$m_t + b_t + \tau_t = \frac{m_{t-1}}{\pi_t} + R_{t-1} \frac{b_{t-1}}{\pi_t}, \quad (16)$$

subject to the transversality condition

$$\lim_{j \rightarrow \infty} \prod_{k=0}^j \left( \frac{\pi_{t+k}}{R_{t+k-1}} \right) b_{t+j} = 0. \quad (17)$$

*Fiscal policy* takes the linear tax rule as in Leeper (1991)

$$\tau_t = \gamma_0 + \gamma b_{t-1} + \varepsilon_t^\tau, \quad (18)$$

where  $\varepsilon_t^\tau$  is an exogenous random shock that is assumed to be i.i.d. with mean zero, and  $0 \leq \gamma \leq \beta^{-1}$ . We follow Leeper (1991) to define fiscal policy as being **active** if  $\gamma < \beta^{-1} - 1$  (AFP) and **passive** if  $\gamma > \beta^{-1} - 1$  (PFP).

We follow Benhabib et al. (2001) and Evans and Honkapohja (2005) with a global non-linear interest rate rule

$$R_t - 1 = \varepsilon_t^R f(\pi_t) \quad (\textit{Taylor rule}), \quad (19)$$

with  $f(\pi)$  assumed to be **non-negative and nondecreasing**, and  $\varepsilon_t^R$  is an exogenous, i.i.d. and positive random shock with a mean of one:

$$f(\pi_t) = (R^* - 1) \left( \frac{\pi_t}{\pi^*} \right)^{\frac{AR^*}{R^* - 1}}, \quad (20)$$

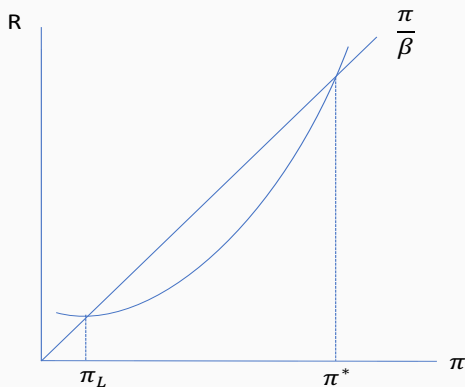
where  $A > 1$ , and  $\pi^* > 1$  is the inflation target.

## Steady states

The Taylor rule (18) implies **two steady states** at the intersection with the Euler/Fisher equation

$$\frac{\pi}{\beta} = 1 + (R^* - 1) \left( \frac{\pi}{\pi^*} \right)^{\frac{AR^*}{R^* - 1}}. \quad (21)$$

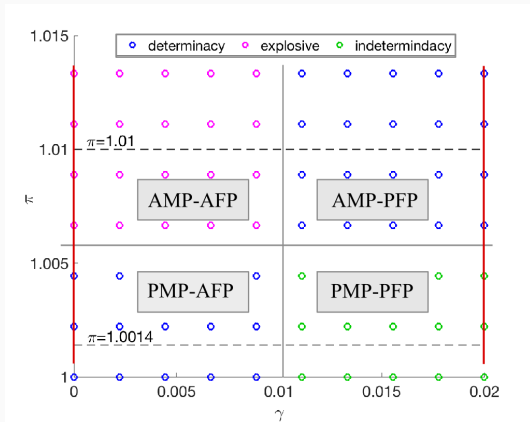
Monetary policy (MP) is said to be **active** at  $\pi^*$  ( $f'(\pi_t) > 1$ ; AMP) and **passive** at  $\pi_L$  ( $f'(\pi_t) < 1$ ; PMP).



This situation is very general and commonly investigated in learning in macroeconomics.

# Policy regimes

Using a standard parameterisation and local stability analysis we obtain four policy regimes



	AMP ( $\pi^*$ )		PMP ( $\pi_L$ )	
	PFP	AFP	PFP	AFP
$\pi_{ss}$	1.0100	1.0100	1.0014	1.0014
$m_{ss}$	1.7157	1.7157	2.0614	2.0614
$c_{ss}/n_{ss}/y_{ss}$	1	1	1	1
$b_{ss}$	4	4	4	4
$u_{ss}$	-1.0170	-1.0170	-1.0118	-1.0118
$\gamma_0$	-0.0566	0.0234	-0.0426	0.0375

# Joining the model and RL

## State representation

$$s_t = \left( m_{t-1}, b_{t-1}, \pi_{t-1}, c_{t-1}, n_{t-1}, \epsilon_t^\tau, \epsilon_t^R, \epsilon_t^y \right). \quad (22)$$

## Household agent actions

$$a_t = (c_t^{act}, b_t^{act}, n_t), \quad (23)$$

where  $x_t^{act} = X_t/P_{t-1}$ ,  $x \in \{c, b\}$ . Information flow and market clearing

$$\pi_t = c_t^{act} / y_t, \quad (24)$$

$$c_t = c_t^{act} / \pi_t, \quad (25)$$

$$b_t = b_t^{act} / \pi_t. \quad (26)$$

**Model environment:** Production, market clearing, pricing, GBC, FP, MP.

**No** first-order conditions (FOC)

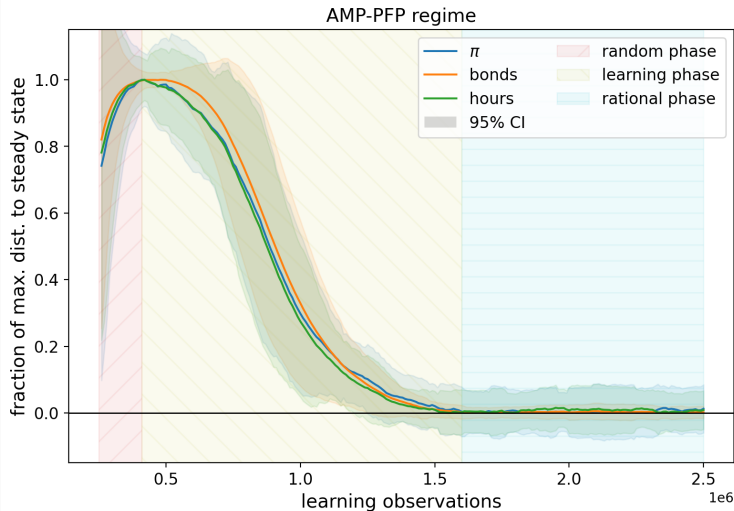
## State transition

1. Observe state  $s_t$
2. Take actions  $\mathcal{P}_\phi(s_t) = a_t = (b_t^{act}, c_t^{act}, n_t)$
3. Production (10) takes place and firm sets wages (12)
4. Markets clear: Inflation  $\pi_t$  is set by (23)
5. This determines real consumption  $c_t$  and bond holdings  $b_t$  (24)-(25)
6. Policy realisations:
  - The monetary authority sets the current gross interest rate  $R_t$  via the Taylor rule (18)
  - The government raises taxes  $\tau_t$  (17)
7. The money holdings  $m_t$  are realised from the GBC (15)
8. Agent obtains reward  $r_t = U(c_t, m_t, n_t)$
9. Next periods shocks are realised,  $(\epsilon_{t+1}^\tau, \epsilon_{t+1}^R, \epsilon_{t+1}^y)$
10. State update  $s_t \leftarrow s_{t+1} = (m_t, b_t, \pi_t, c_t, n_t, \epsilon_{t+1}^\tau, \epsilon_{t+1}^R, \epsilon_{t+1}^y)$

## Results

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# Steady state learning in the AMP-PFP regime

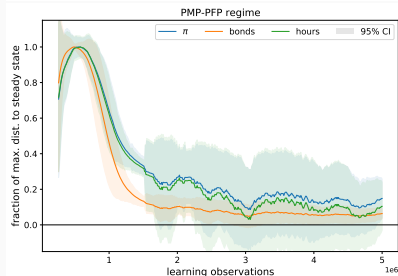
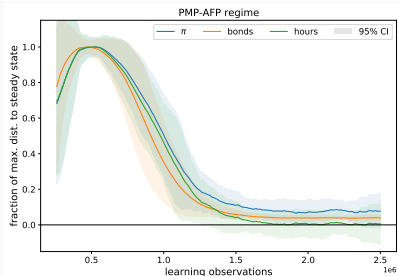
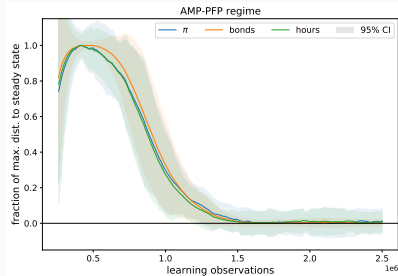
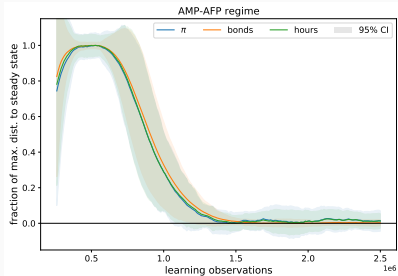


## Learning phases

- random (agent initialisation)
- learning
- rational



# Steady state learning in all regimes (charts)



## Steady state learning in all regimes (table)

	AMP ( $\pi^*$ )		PMP ( $\pi_L$ )	
	PFP	AFP	PFP	AFP
AL	yes	no	no	yes
RL	yes	yes	yes <sup>†</sup>	yes <sup>†</sup>
	$ \Delta_{ss} $ (%) for RL			
$\pi$	0.346	0.278	9.217	5.209
$b$	0.005	0.004	0.038	0.024
$n$	0.004	0.003	0.009	0.003
$m$	0.091	0.089	11.569	7.364
$u$	0.003	0.003	0.346	0.196

<sup>†</sup>imprecision in learning about inflation at  $\pi_L$ .

## Measuring bounded rationality

The household is said to behave rational if it follows FOC. During learning, we define the **FOC-distance** to measure deviations in a standardised way

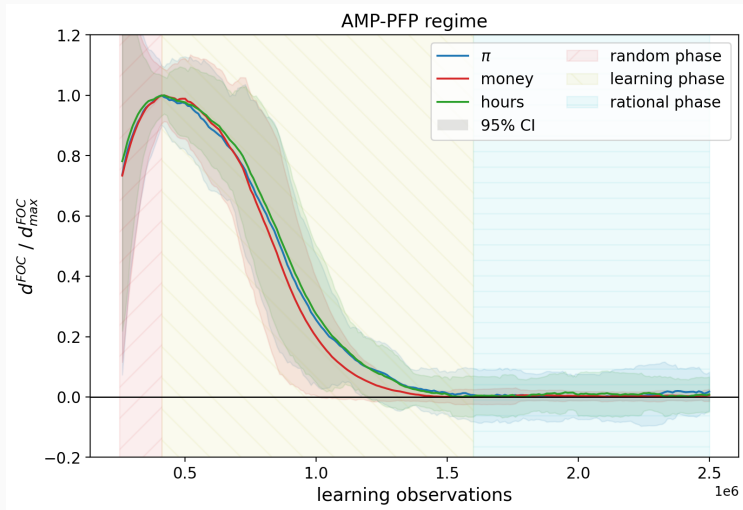
$$d_x^{FOC} \equiv |FOC(x) - 1|, \quad (27)$$

The explicit expression for the **Euler equation**, or *Euler distance*, is

$$d_\pi^{FOC} = \left| \beta \mathbb{E}_t \left[ \left( \frac{c_{t+1}}{c_t} \right)^{-\sigma} \frac{R_t}{\pi_{t+1}} \right] - 1 \right|. \quad (28)$$

FOC distances evaluate agent actions  $\mathcal{P}(s)$ . Analogous measures for  $V/Q$  can be derived with respect to state values.

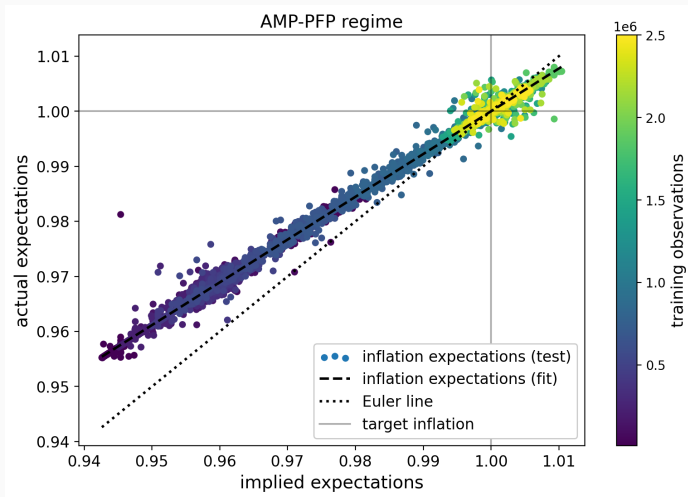
# FOC learning in the AMP-PFP regime



The same **Learning phases** as expected by GPI.

# Measuring inflation expectations during learning

Implied agent expectations can be extracted from realised values



- Improve and better understand learning **robustness**
- Aim for truly global learning
- Compare IRF with those from adaptive learning
- Conduct **experiments**: policy or regimes shifts

## Take-away messages

- DRL offers a **general approach** to solve structural macro models
- **Quantify bounded rationality** and learning in a principled way
- **Global** solution techniques which can also address heterogeneity
- **All policy regimes** are learnable under DRL
- **Promising toolbox** for (macro)economics
- Learning and convergence **technically challenging**

Thanks for listening

Q & A



## Model dynamic properties I

The deterministic steady states in the absence of random shocks is characterised by the following set of equations:

$$\text{Euler / Fisher Equation: } R = \frac{\pi}{\beta} \quad (29)$$

$$\text{Money Demand: } m = y \left( \frac{\pi - \beta}{\chi \pi} \right)^{-1/\sigma} \quad (30)$$

$$\text{Monetary Policy: } R = 1 + (R^* - 1) \left( \frac{\pi}{\pi^*} \right)^{\frac{AR^*}{R^* - 1}} \quad (31)$$

$$\text{Fiscal Policy \& GBC: } b = \left( \frac{1}{\beta} - 1 - \gamma \right)^{-1} [\gamma_0 + (1 - \frac{1}{\pi})m] \quad (32)$$

$$\text{Output: } y^{\sigma+\varphi} = 1 \quad (33)$$

Equation (28) and (30) together determine the steady state of inflation:

$$\frac{\pi}{\beta} = 1 + (R^* - 1) \left( \frac{\pi}{\pi^*} \right)^{\frac{AR^*}{R^* - 1}} \quad (34)$$

## Model dynamic properties II

In the neighbourhood of either steady state, our model can be described by a linear approximation for  $\pi_t$  and  $b_t$  of the form

$$\begin{bmatrix} \hat{\pi}_t \\ \hat{b}_t \end{bmatrix} = \mathbf{B} \begin{bmatrix} \hat{E}_t \pi_{t+1} \\ \hat{E}_t b_{t+1} \end{bmatrix} + \mathbf{C} \begin{bmatrix} \hat{\varepsilon}_t^R \\ \hat{\varepsilon}_t^\tau \\ \hat{\varepsilon}_t^y \end{bmatrix}. \quad (35)$$

**Proposition:**(Evans and Honkapohja, 2007)] In the linear system given by (34),

- (i) If fiscal policy is passive,  $|\gamma - \beta^{-1}| < 1$ , the steady state  $\pi^*$  is locally determinate and the steady state  $\pi_L$  is locally indeterminate.
- (ii) If fiscal policy is active,  $|\gamma - \beta^{-1}| > 1$ , the steady state  $\pi^*$  is locally explosive and the steady state  $\pi_L$  is locally determinate.

# Model parameters

parameter	value	description
$\beta$	0.9900	discount factor
$\sigma$	3.0000	inverse of intertemporal elasticity of consumption and money holdings
$\varphi$	1.0000	inverse of Frisch elasticity of labor supply
$\chi$	0.1000	relative preference weight of money holdings
$\gamma_P$	0.0200	passive fiscal policy (PFP) coefficient
$\gamma_A$	0.0000	active fiscal policy (AFP) coefficient
$A$	1.3000	Taylor rule coefficient
$\pi^*$	1.0100	target gross high-inflation rate (4% net per annum)
$\pi_L$	1.0014	implied gross low-inflation steady state (see Figure ??)
$\epsilon_t^\tau$	0.0005	monetary policy shock (std. dev.)
$\epsilon_t^R$	0.0005	fiscal policy shock (std. dev.)
$\epsilon_t^y$	0.0005	technology shock (std. dev.)

Baseline model parameterisation. The shock series  $\epsilon_t^\tau$ ,  $\epsilon_t^R$ ,  $\epsilon_t^y$  follow log-normal, normal and normal distributions, with means of one, zero and one, respectively.

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