Deep Reinforcement Learning in a Monetary Model

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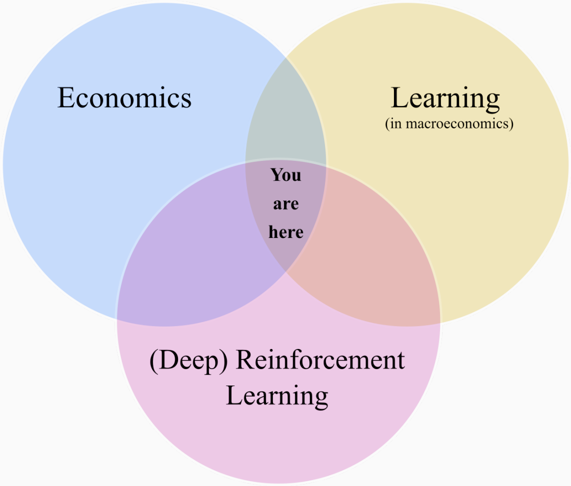
Overview


Economics
Learning
(in macroeconomics)
You
are
here
(Deep) Reinforcement
Learning

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Overview & motivation

Learning in (macro)economics

- Rational Expectation (RE) convenience choice to solve a model, but not necessarily how people and businesses actually behave
- Learning approach to bounded rationality (BR): specify agent knowledge and behaviour away from RE (often ad hoc)
- BR broadens available state space. See also Moll (2024).
- Example: Monetary policy reaction functions possibly very different under learning, such as forward guidance or the stability of Taylor rules

See Eusepi and Preston (2018) and Hommes (2021) for recent reviews.

Example: Adaptive learning

Agents are "econometricians" trying to estimate expected quantities via

$$x^e_{t+1} = x^e_t + \phi_t(x_{t-1} - x^e_t)\,, \tag{1}$$

with a gain series ϕ_t.

Under least-squares learning it is usually taken to be $1/t$. Together with the (optimal) behavioural rules, i.e. linearised FOCs, this leads to a set of ordinary differential equations determining the expectations (E-)stability of the model.

That is, if a steady state is stable under learning, which then serves as a selection criterion.

See, Sargent (1993) and Evans and Honkapohja (2001).

Models populated with *Adaptively Learning Agents* put the agents on an equal footing with the econometrician who is observing data from the model.

- However, this type of parametric recursive method assumes that agents correctly specify the laws of motion and other relevant functional relationships of the model

We work with models populated by *Deep Reinforcement Learning Agents (a.k.a. Artificially Intelligent Agents)* who

- have no a priori knowledge about the structure of the economy
- use their utility realisations in response to their actions in order to learn nonlinear decision rules via deep artificial neural networks

We adopt a policy-based deep reinforcement learning approach that can deal with high dimensional continuous action spaces.

Our approach enables agents to learn flexibly, as our learning algorithms are nonparametric and recursive, reducing the risk of misspecification

Allowing for misspecification and learning via expelling rational expectation agents and replacing them with "artificially intelligent" ones is also **reminiscent Sargent (1993)**

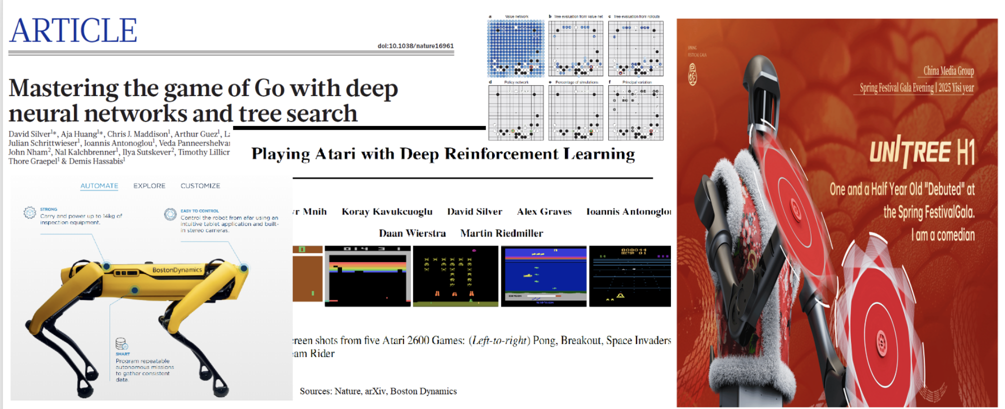
Applications of Deep (Reinforcement) Learning in macroeconomics

- Global solution technique with no need of linearisation or other approximations
- Principled way to (bounded) rationality, i.e. agent behaviour and knowledge (this paper)
- General approach to heterogeneity, e.g. household income or age distribution (Hill et al., 2021).

$\Rightarrow$ Loads of potential applications in (macro)economics!

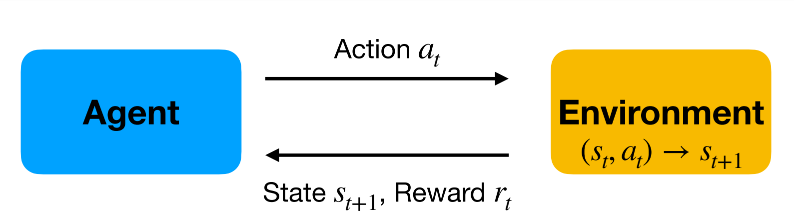
Deep Reinforcement Learning (a.k.a. Artificial Intelligence)

DRL at centre of recent advances in Artificial Intelligence



Sources: Nature, arXiv, Boston Dynamics

The Reinforcement Learning (RL) setting



1. Agent observes state of the world s_t
2. Agent takes actions $a_t(s_t)$
3. Agent receives reward r_t from environment
4. Actions and state lead to state transition of the environment s_{t+1}

This setting is very general. See Sutton and Barto (2018) for a comprehensive introduction.

Formal RL definition

The agent aims to maximise expected cumulative lifetime reward, or expected return,

$$\max_{\mathcal{P}} \mathbb{E}_t[G_t] \quad \text{with} \quad G_t \equiv \sum_{k=0}^{\infty} \beta^k r_{t+1+k}(s)\,, \tag{2}$$

following a behavioural policy $\mathcal{P} : s_t \rightarrow a_t$, with $s_t \in \mathcal{S} \subset \mathbb{R}^{n_s}$ (state space) and $a_t \in \mathcal{A} \subset \mathbb{R}^{n_a}$ (action space).

The environment the agents interaction with returns a reward and a new state, i.e. $\mathcal{E} : (s_t, a_t) \rightarrow (s_{t+1}, r_t)$, with $r_t \in \mathbb{R}$.

The state transitions is modelled as a Markov decision process (MDP) $\mathcal{T} : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \rightarrow Pr(s_{t+1}|s_t, a_t) \in [0, 1]$.

Problem: Writing down $\mathcal{T}$ is simple, knowing $\mathbb{E}_t[G_t]$ and $Pr(s_{t+1}|s_t, a_t)$ is hard (dynamic programming, value function iteration, etc.).

State and action values

The expected return is maximised by finding the policy $\mathcal{P}^*$, which maximises the values function

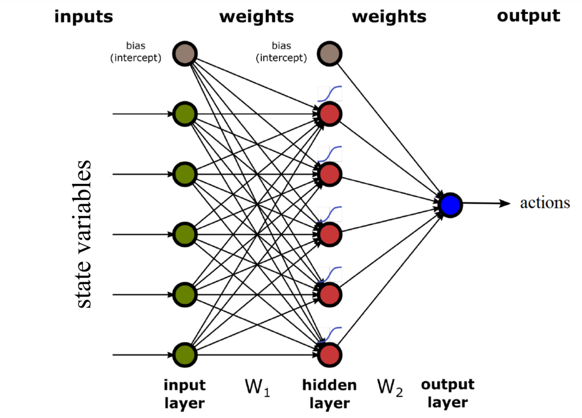
$$
\begin{aligned}
V_{\mathcal{P}}(s) &= \max_{a \in \mathcal{A}} \mathbb{E}_{\mathcal{P}}\left[G_t | s = s_t, a = a_t\right] && (3) \\
&= \max_{a \in \mathcal{A}} Q(s, a)\,, && (4)
\end{aligned}
$$

with $Q(s, a)$ the state action value function. We are done if we know $\mathcal{P}^*$ and V^*/Q^*.

There are different ways to address this problem, which is an area of active AI research.

Deep Learning + Reinforcement Learning = DRL

In DRL, functions $\mathcal{P}$ and V/Q are parameterised using deep artificial neural networks (Goodfellow et al., 2016), i.e. neural nets with several hidden layers, $\mathcal{P}_\phi$ and Q_θ:



Actor-critic DRL setting

$\mathcal{P}$ and Q fulfil the Bellman equation

$$Q(s_t, a_t) = r(s_t) + \beta \mathbb{E}_{\mathcal{P}} \left[Q(s_{t+1}, a_{t+1}) \right]. \tag{5}$$

using sampled state transitions as observations, i.e. interactions of the agent and the environment, and standard optimisation techniques like stochastic gradient descent, the policy and action-value function networks can be trained by iteratively minimising the Bellman residuum,

$$\begin{aligned} L(\phi, \theta) &= \mathbb{E}_{s_t, a_t, r_t} \left[\frac{1}{2} \left(Q_\theta(s_t, a_t) - \hat{Q}_\theta(s_t, a_t) \right)^2 \right], & (6) \\ \text{with} \quad \hat{Q}_\theta(s_t, a_t) &= r_t(a_t, s_t) + \beta \, \mathbb{E}_{\mathcal{P}} \left[Q_\theta \left(s_{t+1}, \mathcal{P}_\phi(s_{t+1}) \right) \right]. & (7) \end{aligned}$$

We use Haarnoja et al. (2018). The code we used for optimisation is available at `https://github.com/pranz24/pytorch-soft-actor-critic`.

General DRL setting for (macro)economics

- Write down model (environment and state)
- Specify learning agents, e.g. households, firms, etc., and their actions
- Specify state transitions as MDP
- Learning using DRL algorithm (e.g. Haarnoja et al. (2018)):
  1. sample state transition(s) and store in memory
  2. train $\mathcal{P}_\phi$ and Q_θ from memory
  3. test $\mathcal{P}_\phi$ and Q_θ with new state transitions and metric of choice

Household learning protocol

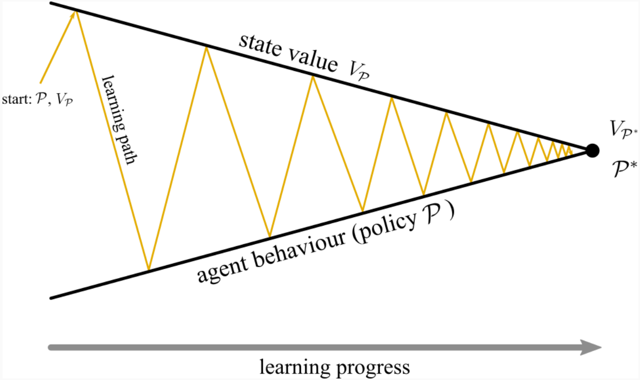
Algorithm 1 Training and testing protocol of household agent

Initialise: Environment $\mathcal{E}$ (parameterised model), agent (parameterised by $\mathcal{P}_\phi$, Q_θ)

for steps = 1 to N_{train} **do**
  initialise training episode with random state s_t
  while training episode is not done **do**
    if steps $\leq N_{burn}$ **then**
      Take allowed random action a_t
    else
      Draw exploration action $a_t = \mathcal{P}_\phi^{exp}(s_t)$
    end if
    Environment returns $(r_t, s_{t+1}) = \mathcal{E}(s_t, a_t)$
    Add transition (s_t, a_t, r_t, s_{t+1}) to memory
    Update $\mathcal{P}_\phi$, Q_θ using batch gradient descent from memory
    if $mod(\text{steps}, N_{interval}) = 0$ **then**
      for test episode = 1 to N_{test} **do**
        Record state transitions (*)
      end for
      Save current agent $(\mathcal{P}_\phi^{steps}, Q_\theta^{steps})$
    end if
    State update $s_t \leftarrow s_{t+1}$
    Test episode termination criteria $(N_{epi}^{max}, d_u^{min})$
  end while
end for
Save final agent $(\mathcal{P}_\phi^{final}, Q_\theta^{final})$

Generalised policy iteration (GPI)

GPI connects economics and learning, and conventional learning approaches with RL



V^*: steady state values, $\mathcal{P}^*$: FOC.

Examples of RL in economics and finance - a very selective literature

- Charpentier et al. (2020): Brief introduction to RL in a economics and finance background
- Zheng et al. (2020): Learning in large-scale geographic ABM
- Calvano et al. (2020): Investigate algorithmic collusion in financial markets
- Chaudhry and Oh (2020): Extract high-frequency expectations in financial markets to measure information effects
- Castro et al. (2021): Learn policy rules of banks participating in a high-value payments system

The Model Environment

Households

A single representative household maximises its expected lifetime utility, subject to an inter-temporal budget constraint:

$$\max_{c_t, m_t, n_t} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, m_t, n_t) \text{ s.t.} \tag{8}$$

$$M_t + B_t + C_t = M_{t-1} + B_{t-1} R_{t-1} + W_t n_t - P_t \tau_t, \tag{9}$$

with P_t the price level at time t, $x_t = \frac{X_t}{P_t}$, $x \in \{M_t, B_t, C_t, W_t\}$ relate real and nominal money, government bonds, consumption and wages, and τ_t is a real lump-sum tax to the government each period.

We take the utility (Evans and Honkapohja, 2005)

$$U(c_t, m_t, n_t) = \frac{c_t^{1-\sigma}}{1-\sigma} + \chi \frac{m_t^{1-\sigma}}{1-\sigma} - \frac{n_t^{1+\varphi}}{1+\varphi}. \tag{10}$$

Firms

A single representative firm produces according to

$$y_t = \varepsilon_t^y n_t , \tag{11}$$

with technology (shock) ε_t^y, maximising profits

$$\max_{w_t} \; y_t - w_t n_t \; , \tag{12}$$

by setting setting the optimal wage

$$w_t = \varepsilon_t^y \; . \tag{13}$$

Market clearing

Markets clear every period, i.e.

$$y_t \;=\; c_t \qquad (goods)\,, \tag{14}$$

and

$$c_t^{\sigma} n_t^{\varphi} = \varepsilon_t^{y} \qquad (labour). \tag{15}$$

Government

The government issues interest-bearing bonds and non-interesting bearing currency (money), and collects taxes under the real inter-temporal *government budget constraint* (GBC)

$$m_t + b_t + \tau_t = \frac{m_{t-1}}{\pi_t} + R_{t-1}\frac{b_{t-1}}{\pi_t}\,, \tag{16}$$

subject to the transversality condition

$$\lim_{j\to\infty}\prod_{k=0}^{j}\left(\frac{\pi_{t+k}}{R_{t+k-1}}\right)b_{t+j} = 0\,. \tag{17}$$

Fiscal policy takes the linear tax rule as in Leeper (1991)

$$\tau_t = \gamma_0 + \gamma b_{t-1} + \varepsilon_t^{\tau}\,, \tag{18}$$

where ε_t^{τ} is an exogenous random shock that is assumed to be i.i.d. with mean zero, and $0 \leq \gamma \leq \beta^{-1}$. We follow Leeper (1991) to define fiscal policy as being active if $\gamma < \beta^{-1} - 1$ (AFP) and passive if $\gamma > \beta^{-1} - 1$ (PFP).

Central bank

We follow Benhabib et al. (2001) and Evans and Honkapohja (2005) with a global non-linear interest rate rule

$$R_t - 1 = \varepsilon_t^R f(\pi_t) \qquad (\textit{Taylor rule}), \tag{19}$$

with $f(\pi)$ assumed to be non-negative and nondecreasing, and ε_t^R is an exogenous, i.i.d. and positive random shock with a mean of one:

$$f(\pi_t) = (R^* - 1)(\frac{\pi_t}{\pi^*})^{\frac{AR^*}{R^*-1}}, \tag{20}$$

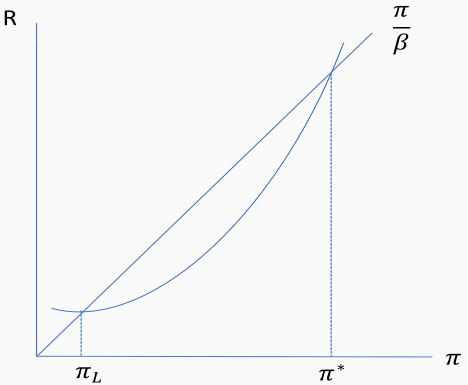
where $A > 1$, and $\pi^* > 1$ is the inflation target.

Steady states

The Taylor rule (19) implies two steady states at the intersection with the Euler/Fisher equation

$$\frac{\pi}{\beta} = 1 + (R^* - 1)(\frac{\pi}{\pi^*})^{\frac{AR^*}{R^*-1}} . \qquad (21)$$

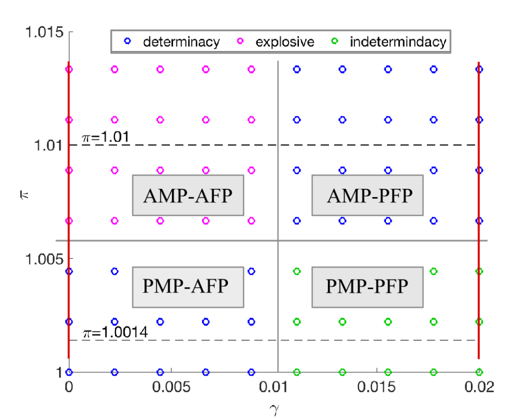
Monetary policy (MP) is said to be active at π^* ($f'(\pi_t) > 1$; AMP) and passive at π_L ($f'(\pi_t) < 1$; PMP).



This situation is very general and commonly investigated in learning in macroeconomics.

Policy regimes

Using a standard parameterisation and local stability analysis we obtain four policy regimes



	AMP (π^*)		PMP (π_L)	
	PFP	AFP	PFP	AFP
π_{ss}	1.0100	1.0100	1.0014	1.0014
m_{ss}	1.7157	1.7157	2.0614	2.0614
$c_{ss}/n_{ss}/y_{ss}$	1	1	1	1
b_{ss}	4	4	4	4
u_{ss}	-1.0170	-1.0170	-1.0118	-1.0118
γ_0	-0.0566	0.0234	-0.0426	0.0375

Joining the model and RL

State representation

$$s_t = \left(m_{t-1}, b_{t-1}, \pi_{t-1}, c_{t-1}, n_{t-1}, \epsilon_t^{\tau}, \epsilon_t^{R}, \epsilon_t^{y} \right) . \tag{22}$$

Household agent actions

$$a_t = \left(c_t^{act}, b_t^{act}, n_t \right) , \tag{23}$$

where $x_t^{act} = X_t / P_{t-1}$, $x \in \{c, b\}$. Information flow and market clearing

$$\pi_t = c_t^{act} / y_t , \tag{24}$$

$$c_t = c_t^{act} / \pi_t , \tag{25}$$

$$b_t = b_t^{act} / \pi_t . \tag{26}$$

Model environment: Production, market clearing, pricing, GBC, FP, MP.

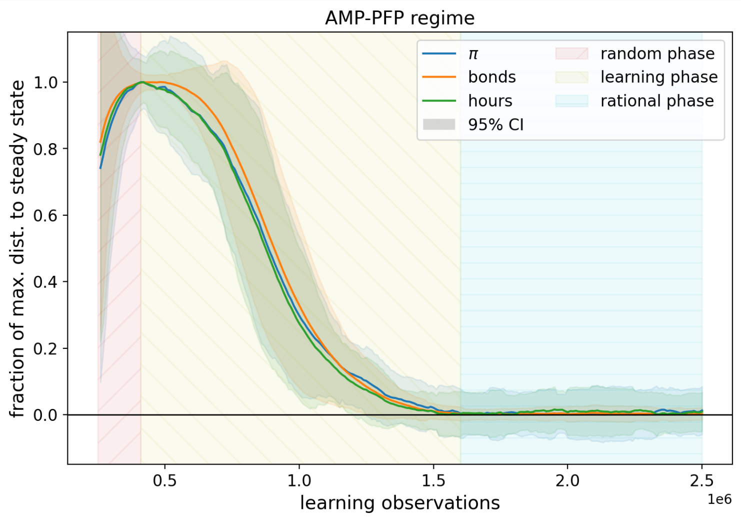
No first-order conditions (FOC)

State transition

1. Observe state s_t
2. Take actions $\mathcal{P}_\phi(s_t) = a_t = (b_t^{act}, c_t^{act}, n_t)$
3. Production (11) takes place and firm sets wages (13)
4. Markets clear: Inflation π_t is set by (24)
5. This determines real consumption c_t and bond holdings b_t (25)-(26)
6. Policy realisations:
    - The monetary authority sets the current gross interest rate R_t via the Taylor rule (19)
    - The government raises taxes τ_t (18)
7. The money holdings m_t are realised from the GBC (16)
8. Agent obtains reward $r_t = U(c_t, m_t, n_t)$
9. Next periods shocks are realised, $(\epsilon^\tau_{t+1}, \epsilon^R_{t+1}, \epsilon^y_{t+1})$
10. State update $s_t \leftarrow s_{t+1} = \left(m_t, b_t, \pi_t, c_t, n_t, \epsilon^\tau_{t+1}, \epsilon^R_{t+1}, \epsilon^y_{t+1}\right)$

Results

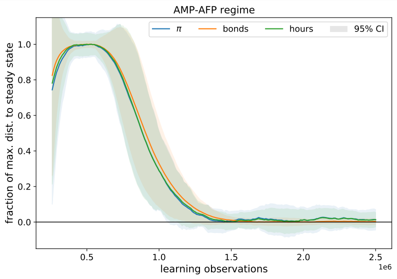
Steady state learning in the AMP-PFP regime

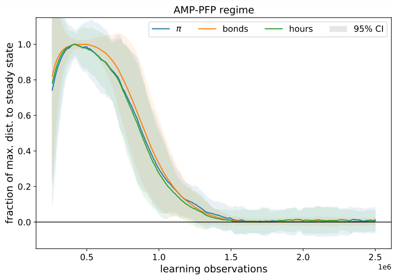


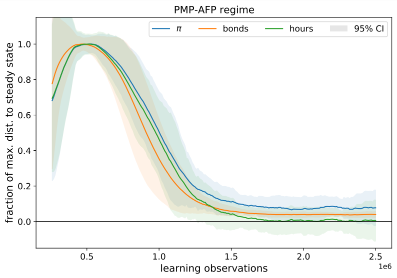
Learning phases

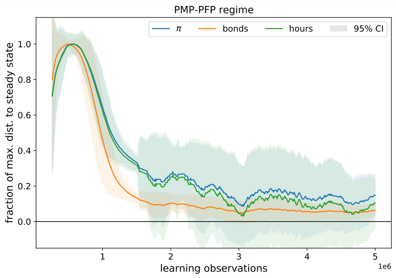
- random (agent initialisation)
- learning
- rational

Steady state learning in all regimes (charts)









Steady state learning in all regimes (table)

	AMP (π^*)		PMP (π_L)	
	PFP	AFP	PFP	AFP
AL	yes	no	no	yes
RL	yes	yes	yes†	yes†
	$\lvert\Delta_{ss}\rvert$ (%) for RL			
π	0.346	0.278	9.217	5.209
b	0.005	0.004	0.038	0.024
n	0.004	0.003	0.009	0.003
m	0.091	0.089	11.569	7.364
u	0.003	0.003	0.346	0.196

†imprecision in learning about inflation at π_L.

Measuring bounded rationality

The household is said to behave rational if it follows FOC. During learning, we define the FOC-distance to measure deviations in a standardised way

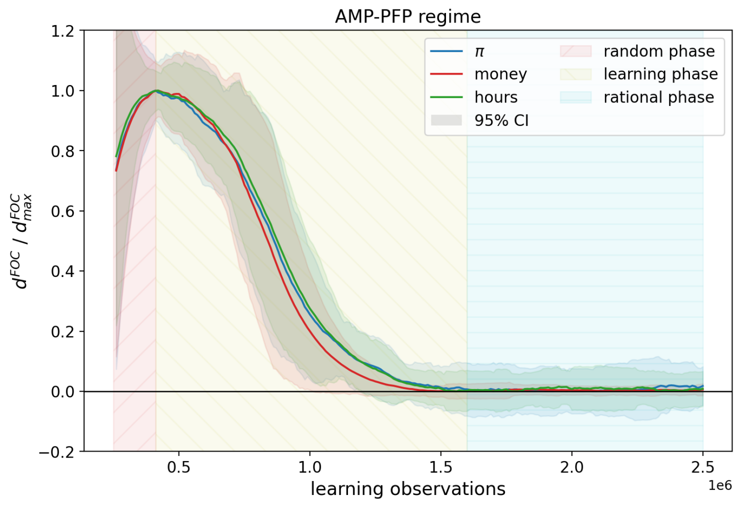
$$d_x^{FOC} \equiv \left| FOC(x) - 1 \right|, \tag{27}$$

The explicit expression for the Euler equation, or *Euler distance*, is

$$d_\pi^{FOC} = \left| \beta\, \mathbb{E}_t \Big[\Big(\frac{c_{t+1}}{c_t}\Big)^{-\sigma} \frac{R_t}{\pi_{t+1}} \Big] - 1 \right|. \tag{28}$$

FOC distances evaluate agent actions $\mathcal{P}(s)$. Analogous measures for V/Q can be derived with respect to state values.

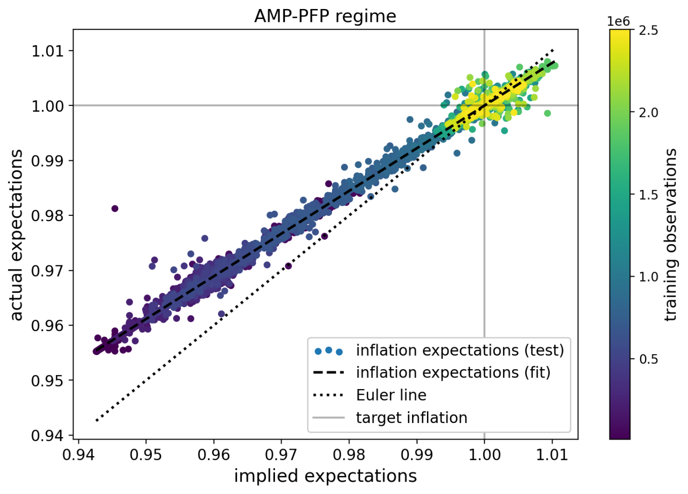
FOC learning in the AMP-PFP regime



The same Learning phases as expected by GPI.

Measuring inflation expectations during learning

Implied agent expectations can be extracted from realised values



Future steps

- Improve and better understand learning robustness
- Aim for truly global learning
- Compare IRF with those from adaptive learning
- Conduct experiments: policy or regimes shifts

Take-away messages

- DRL offers a general approach to solve structural macro models
- Quantify bounded rationality and learning in a principled way
- Global solution techniques which can also address heterogeneity
- All policy regimes are learnable under DRL
- Promising toolbox for (macro)economics
- Learning and convergence technically challenging

Thanks for listening

Q & A

Model dynamic properties I

The deterministic steady states in the absence of random shocks is characterised by the following set of equations:

$$\text{Euler / Fisher Equation:} \quad R = \frac{\pi}{\beta} \tag{29}$$

$$\text{Money Demand:} \quad m = y\left(\frac{\pi - \beta}{\chi\pi}\right)^{-1/\sigma} \tag{30}$$

$$\text{Monetary Policy:} \quad R = 1 + (R^* - 1)\left(\frac{\pi}{\pi^*}\right)^{\frac{AR^*}{R^*-1}} \tag{31}$$

$$\text{Fiscal Policy \& GBC:} \quad b = \left(\frac{1}{\beta} - 1 - \gamma\right)^{-1}\left[\gamma_0 + \left(1 - \frac{1}{\pi}\right)m\right] \tag{32}$$

$$\text{Output:} \quad y^{\sigma+\varphi} = 1 \tag{33}$$

Equation (29) and (31) together determine the steady state of inflation:

$$\frac{\pi}{\beta} = 1 + (R^* - 1)\left(\frac{\pi}{\pi^*}\right)^{\frac{AR^*}{R^*-1}} \tag{34}$$

Model dynamic properties II

In the neighbourhood of either steady state, our model can be described by a linear approximation for π_t and b_t of the form

$$\begin{bmatrix} \hat{\pi}_t \\ \hat{b}_t \end{bmatrix} = \mathbf{B} \begin{bmatrix} \hat{E}_t \pi_{t+1} \\ \hat{E}_t b_{t+1} \end{bmatrix} + \mathbf{C} \begin{bmatrix} \hat{\varepsilon}_t^R \\ \hat{\varepsilon}_t^\tau \\ \hat{\varepsilon}_t^y \end{bmatrix}. \tag{35}$$

Proposition:(Evans and Honkapohja, 2007)] In the linear system given by (35),
(i) If fiscal policy is passive, $|\gamma - \beta^{-1}| < 1$, the steady state π^* is locally determinate and the steady state π_L is locally indeterminate.
(ii) If fiscal policy is active, $|\gamma - \beta^{-1}| > 1$, the steady state π^* is locally explosive and the steady state π_L is locally determinate.

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Model parameters

parameter	value	description
β	0.9900	discount factor
σ	3.0000	inverse of intertemporal elasticity of consumption and money holdings
φ	1.0000	inverse of Frisch elasticity of labor supply
χ	0.1000	relative preference weight of money holdings
γ_P	0.0200	passive fiscal policy (PFP) coefficient
γ_A	0.0000	active fiscal policy (AFP) coefficient
A	1.3000	Taylor rule coefficient
π^*	1.0100	target gross high-inflation rate (4% net per annum)
π_L	1.0014	implied gross low-inflation steady state (see Figure **??**)
ϵ_t^τ	0.0005	monetary policy shock (std. dev.)
ϵ_t^R	0.0005	fiscal policy shock (std. dev.)
ϵ_t^y	0.0005	technology shock (std. dev.)

Baseline model parameterisation. The shock series ϵ_t^τ, ϵ_t^R, ϵ_t^y follow log-normal, normal and normal distributions, with means of one, zero and one, respectively.

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