### An Introduction to Deep Reinforcement Learning

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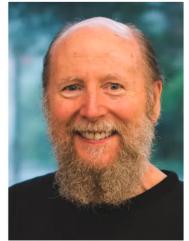
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#### Overview

- ► This talk introduces Deep Reinforcement Learning (DRL), an important branch of modern Artificial Intelligence (AI).
- We will examine the fundamental concepts and core principles of Reinforcement Learning (RL).
- The close relationship between RL and Dynamic Programming (DP)—a foundational tool in economics—will be emphasized.
- ▶ We will explore how DRL can be applied in economics, finance, and complex decision-making problems.
- Our goal is to inspire students to consider applying DRL in economic modeling and related research.

#### The Recent News...





Andrew Barto (left) and Richard Sutton (right) have won the 2024 Turing Award. Image: Association for Computing Machinery

Figure: Al pioneers Andrew Barto and Richard Sutton win 2025 Turing Award for groundbreaking contributions to reinforcement learning

#### What is Artificial Intelligence?

- ► Al is the study of how to make computers **think and learn like humans**.
- It combines ideas from computer science, math, statistics, and cognitive science.
- Goal: Build systems that can solve problems, learn from data, and adapt to new situations.

#### Simple Examples:

- ► Siri, Alexa (voice assistants)
- Spam filters in email
- Netflix recommendations

### Types of Al

- ► Narrow AI: Designed for specific tasks (e.g., playing chess, recognizing faces)
- General AI: Human-level intelligence across a wide range of tasks (still theoretical)
- ➤ **Superintelligent AI**: Hypothetical AI that surpasses human intelligence (philosophical/futuristic)

Today, almost all AI is Narrow AI.

### Key Techniques in Al

- ► Machine Learning (ML) Letting computers learn patterns from data.
- ▶ Deep Learning A powerful type of ML using neural networks with many layers.
- Natural Language Processing (NLP) − Enabling machines to understand and generate human language.
- Computer Vision Teaching machines to "see" and interpret images/videos.

### Al in Everyday Life

- ► Google Maps (route prediction)
- Spotify/Netflix (recommendations)
- Online shopping (personalized ads)
- Face ID (facial recognition)
- Self-driving cars

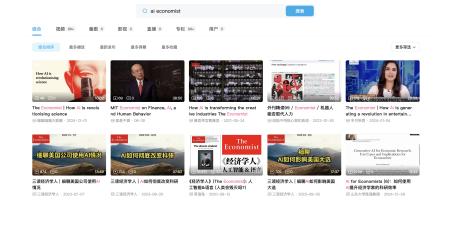
Al is already everywhere — often behind the scenes!

#### Challenges and Ethics in Al

- Bias in algorithms
- Privacy and surveillance
- Job displacement and automation
- Accountability and transparency

Al is powerful — but we must use it responsibly.

#### Al Economist



### Large Language Model

- ► A type of artificial intelligence trained to understand and generate human language.
- ▶ Based on deep learning, especially transformer architecture (e.g., GPT, BERT).
- ► Trained on vast amounts of text data to learn grammar, facts, reasoning patterns, and more.

#### **Key Features**

- ▶ Predicts the next word in a sentence (language modeling).
- ► Can perform tasks like translation, summarization, question answering, and conversation.
- ► Learns *statistical patterns* in language—not conscious or understanding like humans.

# **Famous Examples:** ChatGPT, Claude, Gemini, LLaMA, Grok, Deepseek

### **Deep Reinforcement Learning**

#### Deep Reinforcement Learning in this Talk

#### ► Fundamental Concepts of Reinforcement Learning

- Definitions and illustrative examples
- Connections between Dynamic Programming and RL
- Comparing RL with Supervised and Unsupervised Learning

#### Advanced Topics and Applications

- Applications of RL in economics
- Non-stationary and non-homogeneous environments in RL
- Multi-agent reinforcement learning
- Distributional reinforcement learning

#### What is Reinforcement Learning?

- Wikipedia: "Reinforcement learning is an interdisciplinary area of machine learning and optimal control concerned with how an intelligent agent should take actions in a dynamic environment in order to maximize a reward signal."
- ► Literal Decomposition:
  - ► Reinforcement: reward-driven
  - Learning: (optimal) policy function
- Components:
  - State of the Environment
  - Action taken by the Agent
  - Reward as a sequence of the State and the Action
- Suitable for problems involving sequential decisions under uncertainty.

### What is Reinforcement Learning? (Cont.)

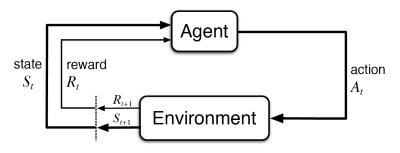


Figure: Agent-Envrionement Interaction by Sutton and Barto (2018)

#### What is RL: Mathematical Definition

- Definition: A Markov decision process (MDP) is a 5-tuple  $(S, A, P, R, \gamma)$ , where:
  - S is a set of states called the state space
  - A is a set of actions called the action space
  - ▶  $P(s, a, s') = \Pr(s_{t+1} = s' \mid s_t = s, a_t = a)$  is the prob. that action a in state s at time t will lead to state s' at time t + 1
  - Arr R(s, a, s') is the immediate reward received after transitioning from state s to state s', due to action a
  - $ightharpoonup \gamma$  is the discount rate

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  - $ightharpoonup \gamma$  is the discount rate
- ► RL solves an MDP problem:
  - ▶ An Agent observes state  $s_t \in \mathcal{S}$ , takes an action  $a_t \in \mathcal{A}$  based on a policy  $g \in \mathcal{S} \to \mathcal{A}$ , the environment produces a reward  $r_t$  and moves to  $s_{t+1}$
  - The goal is to find an optimal policy that obtaining accumulative rewards  $\sum_{i=1}^{n} \gamma^{t} R_{t}$  using a training algorithm

### RL and Dynamic Programming (DP)

- ▶ RL is closely related to DP through the Bellman Equation (which will be introduced later).
- ▶ DP assumes a known environment model; RL learns through interaction when the model is unknown.
- ▶ DP methods: Value Iteration, Policy Iteration
- ► RL methods: **Q-learning**, **Policy Gradient**

**In Economics:** DP is widely used in models like consumption-savings, investment, and dynamic games. RL provides a data-driven alternative.

### What is Deep Reinforcement Learning (DRL)?

- DRL combines Reinforcement Learning with Deep Neural Networks.
- Deep networks approximate policies or value functions useful in high-dimensional settings.
- Breakthroughs in DRL allow AI to operate in complex, real-world environments.

#### Famous Applications:

- AlphaGo (Go playing AI)
- Autonomous driving, robotics
- ► Game AI (Atari, StarCraft)

### DRL at Centre of Recent Advances in Artificial Intelligence

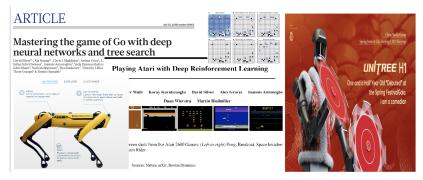


Figure: Caption

#### Applications of DRL in Economics

- Optimal consumption and saving under uncertainty with large state spaces.
- Asset allocation and trading strategies in financial markets.
- Macro policy simulation: Al agents adjusting fiscal/monetary policy in simulated economies.
- Mechanism design and market experiments using simulated agents.

**Strengths:** Model-free, scalable, adaptive to nonlinear and dynamic environments.

### What is RL: Example I

State: current position

Action: Up, Low, Left, Right

► Reward: ?

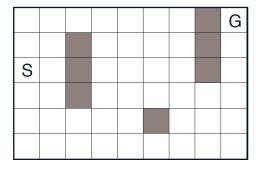


Figure: An Maze Problem

#### What is RL: Example II

➤ The *Frozen-Lake* Environment: "The ice is slippery, so you won't always move in the direction you intend."

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SFFF (S: starting point, safe)
FHFH (F: frozen surface, safe)
FFFH (H: hole, fall to your doom)
HFFG (G: goal, where the frisbee is located)
```

Figure: Frozen-Lake

### What is RL: Example III

- ► The *Cart-Pole* Environment: GIF
- State:
  - ► Cart Position: [-4.8, 4.8]
  - ► Cart Velocity: [-Inf, Inf]
  - ► Pole Angle: [-24°, 24°]
  - ► Pole Angular Velocity: [-Inf, Inf]
- ► Action: 0 (Left) or 1 (Right)
- ▶ Reward: +1 for every step unless failed

### What is RL: Example IV

- A McCall job search model in labor economics
- ► State: features that characterizing an offer, for example,  $(w_t)$  where  $w_t \in [w_{\min}, w_{\max}]$
- ► Action: 0 (Reject) or 1 (Accept)
- Reward: unemployment compensation c if Reject and w<sub>t</sub> if Accept and the game ends

#### What is RL: Example V

- A consumption-saving model (finite or infinite horizon) in macroeconomics
- ▶ State:  $(k_t, \epsilon_t)$ , where  $k_t \in [k_{\min}, k_{\max}]$  is the capital holding,  $\epsilon_t \in \{0, 1\}$  is the employment status
- $\triangleright$  Action:  $c_t$ , the consumption
- ightharpoonup Reward:  $u(c_t)$ , the utility

#### Intuition Behind RL Algorithms

- ▶ Define the accumulative reward  $G_t = \sum_{t=1}^n \gamma^t R_t$
- ► The celebrated Bellman Equation:

$$V^{*}(s) = \max_{a} \mathbb{E} [R_{t} + \gamma G_{t+1} \mid S_{t} = s, A_{t} = a]$$

$$= \max_{a} \mathbb{E} [R_{t} + \gamma V_{*}(S_{t+1}) \mid S_{t} = s, A_{t} = a]$$

$$= \max_{a} R_{t} + \gamma \sum_{s'} P(s'|s, a) V^{*}(s')$$

Version for State-Action Value Function (Q-Function):

$$Q^*(s, a) = R(s, a) + \gamma \sum_{s'} P(s'|s, a) \max_{a'} Q^*(s', a')$$

Note that:

$$g^*(s) = \underset{a}{\operatorname{arg max}} Q^*(s, a)$$
$$V^*(s) = \underset{a}{\operatorname{max}} Q^*(s, a)$$

#### The Agent

- The decision-making policy g:
  - Indirect: value function approach: V(s) or Q(s, a)
  - ▶ Direct: policy function approach: a = g(s)
  - ► How to parameterize the value/policy function?
- ► The behavioral policy:
  - **E**.g., the  $\epsilon$ -greedy policy:

$$\pi(a|s) = \begin{cases} 1 - \epsilon + \frac{\epsilon}{|\mathcal{A}(s)|}, & \text{if } a = \operatorname{argmax}_{a'} Q(s, a') \\ \frac{\epsilon}{|\mathcal{A}(s)|}, & \text{otherwise} \end{cases}$$

- ► The exploration-exploitation trade-off
- ▶ Other structures facilitate the solution: e.g. the "memory for experiences"

#### Machine Learning: SL, UL, RL

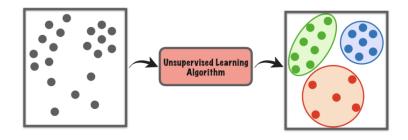
- ► Three broad categories: Supervised Learning (SL), Unsupervised Learning (UL) and Reinforcement Learning (RL)
- ► SL: "You know what is true"
  - ▶ Data:  $\{x_i, y_i\}_{i=1...N}$
  - ▶ Task: find  $f: \mathbb{X} \to \mathbb{Y}$  such that  $f(x) \approx y$
- ▶ UL: "You don't know what is true"
  - ▶ Data:  $\{x_i\}_{i=1...N}$
  - ► Task: find some sort of underlying structure, correctly label/group the data based on *x<sub>i</sub>*
- RL: "You know what shall be true"
  - ▶ Data:  $\{x_t\}_{t=1...T}$  is our generated state,  $\{r_t\}_{i=1...T}$  "signals of correctness"
  - ▶ Task: find  $f: \mathbb{X} \to \mathbb{Y}$  an optimal policy function

### Supervised Learning: An illustration

The "Hello World" problem in supervised learning

Figure: MNIST data

### Unsupervised Learning: An illustration



#### Optimal Control: DP and RL

- ► Recall the Bellman Equation in terms of Q-Function:  $Q_*(s, a) = R(s, a) + \gamma \sum_{s'} P(s'|s, a) \max_{a'} Q_*(s', a')$
- Dynamic Programming (DP): P is known and closed-form
- ► In practice:
  - P and R is not known or hard to express in closed-form
  - $\triangleright$  S, A is continuous/high-dimensional
  - the max operator is computationally expensive
- ▶ Problem 1: Simulation. The celebrated Q-learning algorithm:  $Q^{i+1}(s,a) = (1-\alpha)Q^i(s,a) + \alpha(r+\gamma \max_{a'} Q^i(s',a'))$
- Problem 2 & 3: we use, e.g., Neural Network (Deep RL)
  - ightharpoonup Critic: A Value Network  $Q_{\theta}(s, a)$
  - ightharpoonup Actor: A Policy Network  $g_{\phi}(s)$

#### RL in Economics: Literature

- ▶ DRL in a Monetary Model (Chen, Joseph, Kumhof, Pan and Zhou, 2021)
- ► AI, algorithmic pricing and collusion (Calvano, Calzolari, Denicolo and Pastorello, 2020)
- Al as structural estimation: Deep Blue, Bonanza, and AlphaGo (Igami, 2020)
- RL for Optimization of COVID-19 Mitigation policies (Kompella, Capobianco, Jong, Browne, Fox, Meyers, Wurman and Stone, 2020)
- ► Al-Economist with tax policies (Zheng, Trott, Srinivasa, Naik, Gruesbeck, Parkes and Socher, 2020)
- Multi-agent RL in Cheap Talk (Condorelli and Furlan, 2023)

### Non-stationary/Non-homogeneous RL

- ► We currently assume the MDP is stationary, i.e., time-invariant (no subscript *t*).
- Non-stationarity arises when:
  - A game ends after a finite number of stages (For example, optimality for a finitely-lived agent).
  - ▶ The MDP structure changes due to exogenous or endogenous shocks. For example, (Li et al., 2022): "In an mHealth study aiming to optimize the timing of smartphone-delivered interventions (designed to encourage anti-sedentary behavior among interns), non-stationarity emerges as users gradually habituate to or become overwhelmed by repeated prompts, leading to diminished responsiveness."

### Multi-agent Reinforcement Learning

#### **Definition**

- Extension of DRL to environments with multiple interacting agents
- ▶ Each agent learns its own policy  $\pi_i : \mathcal{O}_i \to \mathcal{A}_i$
- ► Collective behavior emerges from individual learning

#### **Challenges:**

- Non-stationarity
- Credit assignment
- Scalability
- Partial observability

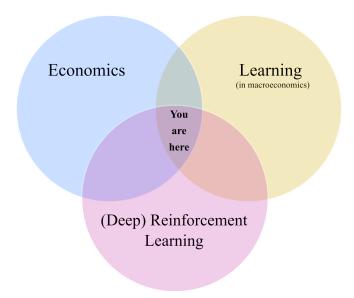
Video for application

#### Approaches:

- Independent DQN
- MADDPG
- QMIX
- MAPPO

## Two Research Projects

### I. Deep Reinforcement Learning in a Monetary Model



# Learning in (macro)economics

- Rational Expectation (RE) convenience choice to solve a model, but not necessarily how people and businesses actually behave
- ► Learning approach to bounded rationality (BR): specify agent knowledge and behaviour away from RE (often ad hoc)
- BR broadens available state space. See also Moll (2024).
- Example: Monetary policy reaction functions possibly very different under learning, such as forward guidance or the stability of Taylor rules

See Eusepi and Preston (2018) and Hommes (2021) for recent reviews.

# Example: Adaptive learning

Agents are "econometricians" trying to estimate expected quantities via

$$x_{t+1}^{e} = x_{t}^{e} + \phi_{t}(x_{t} - x_{t}^{e}),$$
 (1)

with a gain series  $\phi_t$ .

Under least-squares learning it is usually taken to be 1/t. Together with the (optimal) behavioural rules, i.e. linearised FOCs, this leads to a set of ordinary differential equations determining the expectations (E-)stability of the model.

That is, if a steady state is stable under learning, which then serves as a selection criterion.

See, Sargent (1993) and Evans and Honkapohja (2001).

Models populated with *Adaptively Learning Agents* put the agents on an equal footing with the econometrician who is observing data from the model.

► However, this type of parametric recursive method assumes that agents correctly specify the laws of motion and other relevant functional relationships of the model We work with models populated by *Deep Reinforcement Learning Agents (a.k.a. Artificially Intelligent Agents)* who

- have no a priori knowledge about the structure of the economy
- use their utility realisations in response to their actions in order to learn nonlinear decision rules via deep artificial neural networks

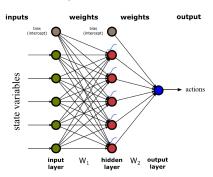
We adopt a policy-based deep reinforcement learning approach that can deal with high dimensional continuous action spaces.

Our approach enables agents to learn flexibly, as our learning algorithms are nonparametric and recursive, reducing the risk of misspecification

Allowing for misspecification and learning via expelling rational expectation agents and replacing them with "artificially intelligent" ones is also **reminiscent Sargent (1993)** 

# Deep Learning + Reinforcement Learning = DRL

In DRL, functions  $\mathcal{P}$  and V/Q are parameterised using deep artificial neural networks (Goodfellow et al., 2016), i.e. neural nets with several hidden layers,  $\mathcal{P}_{\phi}$  and  $Q_{\theta}$ :



# Actor-critic DRL setting

 $\mathcal{P}$  and Q fulfil the Bellman equation

$$Q(s_t, a_t) = r(s_t, a_t) + \beta \mathbb{E}_{\mathcal{P}} [Q(s_{t+1}, a_{t+1})].$$
 (2)

using sampled state transitions as observations, i.e. interactions of the agent and the environment, and standard optimisation techniques like stochastic gradient descent, the policy and action-value function networks can be trained by iteratively minimising the Bellman residuum,

$$L(\phi,\theta) = \mathbb{E}_{s_t,a_t,r_t}\left[\frac{1}{2}(Q_{\theta}(s_t,a_t)-\hat{Q}_{\theta}(s_t,a_t))^2\right],(3)$$

with 
$$\hat{Q}_{\theta}(s_t, a_t) = r(a_t, s_t) + \beta \mathbb{E}_{\mathcal{P}}[Q_{\theta}(s_{t+1}, \mathcal{P}_{\phi}(s_{t+1}))]$$
 (4)

We use Haarnoja et al. (2018). The code we used for optimisation is available at

https://github.com/pranz24/pytorch-soft-actor-critic.

# General DRL setting for (macro)economics

- Write down model (environment and state)
- ➤ Specify learning agents, e.g. households, firms, etc., and their actions
- Specify state transitions as MDP
- ► Learning using DRL algorithm (e.g. Haarnoja et al. (2018)):
  - 1. sample state transition(s) and store in memory
  - 2. train  $\mathcal{P}_{\phi}$  and  $Q_{\theta}$  from memory
  - 3. test  $\mathcal{P}_{\phi}$  and  $\mathcal{Q}_{\theta}$  with new state transitions and metric of choice

# II. Distributional RL (Bellemare et al., 2023)

Consider the case that you need to catch an early morning flight to a conference and want to maximize your sleep by choosing the shortest trip to the airport.

#### Options:

- Routine A: Always takes 1.5 hours.
- ▶ Routine B: Typically takes 1.2 hours, but has a 10% chance of requiring maintenance, which could extend the trip to over 3 hours and result in missing your flight.

#### Decision Trade-off:

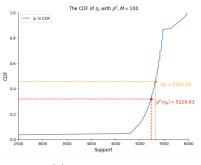
- ► An expectation maximizer would favor Routine B for its shorter average travel time.
- However, if avoiding the risk of missing your flight is a high priority, you might prefer the consistent timing of Routine A.

# Distributional RL (Bellemare et al., 2023)

- ▶  $\eta^*$ : value distribution function, where  $\eta^*(s, a)$  represents the distribution of  $G^*(s, a)$  for a given state-action pair (s, a)
- Our objective is to seek an optimal policy  $\pi^*$  such that for each state s we maximize a generalized value of return measure by a particular  $\rho$ :

$$\pi^*(s) = \arg\max_{a} \rho(\eta^*(s,a))$$

### Distributional RL: The McCall Model



(a) The CDF of  $\eta_r$ 

(b) The policy functions  $\pi_{\mathcal{Q}}$  and  $\pi_{\eta}$ 

#### Conclusion

- ▶ RL is nothing far away from economists
- RL could potentially help us to solve some complex settings where we should rely on simulations to solve agents' decision-makings
- MARL could even go further to study more interactive settings
  - policy-makers' problem in macro
  - strategic plays in game theory
  - firms' interaction in IO
- Distributional RL could help us to take into consideration the agent's risk aversion

# Summary Outlook

- RL offers new ways to approach dynamic decision-making under uncertainty.
- DRL expands the power of RL to handle complex, high-dimensional problems.
- Many economic problems are naturally sequential and benefit from RL techniques.
- Students are encouraged to explore how AI can enhance research in economics.

The economist of the future may also be an AI engineer.

- Bellemare, Marc G, Will Dabney, and Mark Rowland, Distributional reinforcement learning, MIT Press, 2023.
- Calvano, Emilio, Giacomo Calzolari, Vincenzo Denicolo, and Sergio Pastorello, "Artificial intelligence, algorithmic pricing, and collusion," *American Economic Review*, 2020, *110* (10), 3267–3297.
- Chen, Mingli, Andreas Joseph, Michael Kumhof, Xinlei Pan, and Xuan Zhou, "Deep reinforcement learning in a monetary model," arXiv preprint arXiv:2104.09368, 2021.
- **Condorelli, Daniele and Massimiliano Furlan**, "Cheap Talking Algorithms," *arXiv preprint arXiv:2310.07867*, 2023.
- **Eusepi, Stefano and Bruce Preston**, "The Science of Monetary Policy: An Imperfect Knowledge Perspective," *Journal of Economic Literature*, March 2018, *56* (1), 3–59.
- **Evans, George W. and Seppo Honkapohja**, *Learning and Expectations in Macroeconomics*, Princeton University Press, 2001.
- Goodfellow, Ian, Yoshua Bengio, Aaron Courville, and Yoshua Bengio, Deep learning, Vol. 1, MIT press Cambridge, 2016.

- Haarnoja, Tuomas, Aurick Zhou, Pieter Abbeel, and Sergey Levine, "Soft Actor-Critic: Off-Policy Maximum Entropy Deep Reinforcement Learning with a Stochastic Actor," *arXiv-eprint*, 2018, *1801.01290*.
- Hommes, Cars, "Behavioral and Experimental Macroeconomics and Policy Analysis: A Complex Systems Approach," *Journal of Economic Literature*, March 2021, *59* (1), 149–219.
- **Igami, Mitsuru**, "Artificial intelligence as structural estimation: Deep Blue, Bonanza, and AlphaGo," *The Econometrics Journal*, 2020, 23 (3), S1–S24.
- Kompella, Varun, Roberto Capobianco, Stacy Jong, Jonathan Browne, Spencer Fox, Lauren Meyers, Peter Wurman, and Peter Stone, "Reinforcement learning for optimization of COVID-19 mitigation policies," arXiv preprint arXiv:2010.10560, 2020.
- Li, Mengbing, Chengchun Shi, Zhenke Wu, and Piotr Fryzlewicz, "Testing stationarity and change point detection in reinforcement learning," arXiv preprint arXiv:2203.01707, 2022.

- Moll, Benjamin, "The Trouble with Rational Expectations in Heterogeneous Agent Models: A Challenge for Macroeconomics," London School of Economics, mimeo, available at https://benjaminmoll. com, 2024.
- **Sargent, Thomas J**, "Bounded rationality in macroeconomics: The Arne Ryde memorial lectures," *OUP Catalogue*, 1993.
- **Sutton, Richard S and Andrew G Barto**, *Reinforcement learning: An introduction*, MIT press, 2018.
- Zheng, Stephan, Alexander Trott, Sunil Srinivasa, Nikhil Naik, Melvin Gruesbeck, David C Parkes, and Richard Socher, "The ai economist: Improving equality and productivity with ai-driven tax policies," arXiv preprint arXiv:2004.13332, 2020.